



EE 232 Lightwave Devices

Lecture 13: Rate Equations and Dynamic Response of Semiconductor Lasers

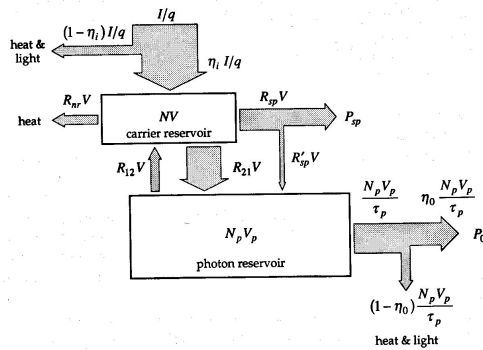
Reading: Chuang, Sec. 11.1 – 11.2
 (See also Coldren, Sec. 5.1 – 5.3)
 (The Notes follow primarily Coldren's book)

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Rate Equations



Rate Equations:

$$\begin{cases} \frac{dN}{dt} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - v_g g(N)S \\ \frac{dS}{dt} = \Gamma v_g g(N)S - \frac{S}{\tau_p} + \Gamma \beta R_{sp} \end{cases}$$

Note that

$$\frac{N}{\tau} = AN + BN^2 + CN^3$$

FIGURE 5.1 Model used in the rate equation analysis of semiconductor lasers.

Coldren, p. 186 Notation in Coldren's Book

$$N_p \leftrightarrow S$$

$$R_{sp} \leftrightarrow BN^2$$

$$R_{nr} \leftrightarrow AN + CN^3$$



Simplified Analysis: Steady State

$$R_{sp} \approx 0$$

Steady State Solution:

$$\frac{dS}{dt} = 0 = \Gamma v_g g(N)S - \frac{S}{\tau_p}$$

$$\Rightarrow g(N) = \frac{1}{\Gamma v_g \tau_p} = \frac{\alpha_{total}}{\Gamma} = g_{th}$$

\Rightarrow Gain "clamped" at threshold

$$\frac{dN}{dt} = 0 = \frac{\eta_i I}{qV} - \frac{N}{\tau} - v_g g(N)S$$

(1) Below Threshold: $S = 0$

$$I = \frac{qNV}{\eta_i \tau}$$

(2) Above Threshold: $S > 0$

$$S = \frac{1}{v_g g_{th}} \frac{\eta_i}{qV} (I - I_{th})$$

$$P_{out} = \frac{h\omega}{\tau_p} \frac{\alpha_m}{\alpha_i + \alpha_m} \frac{VS}{\Gamma}$$

$$= \frac{1}{v_g g_{th}} \frac{h\omega}{\tau_p} \frac{\alpha_m}{\alpha_i + \alpha_m} \frac{\eta_i}{qV} \frac{V}{\Gamma} (I - I_{th})$$

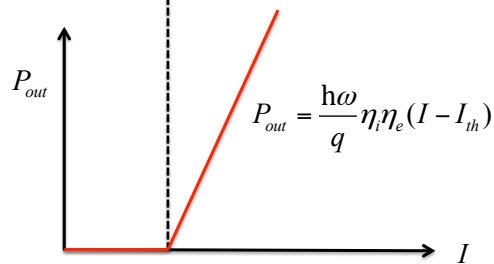
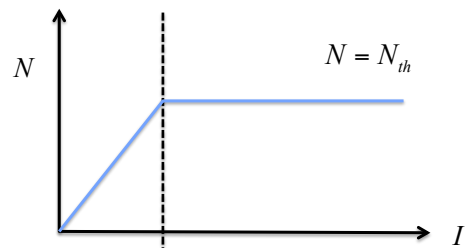
$$P_{out} = \frac{h\omega}{q} \eta_i \eta_e (I - I_{th})$$

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Simplified Analysis: Steady State



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Detailed Analysis: Steady State Solutions

$$\frac{dS}{dt} = 0 = \Gamma v_g g(N)S - \frac{S}{\tau_p} + \Gamma \beta R_{sp}$$

$$\Rightarrow S(N) = \frac{\Gamma \beta R_{sp}(N)}{\frac{1}{\tau_p} - \Gamma v_g g(N)} \quad (1)$$

$$\frac{dN}{dt} = 0 = \frac{\eta_i I}{qV} - \frac{N}{\tau(N)} - v_g g(N)S(N)$$

$$\Rightarrow I(N) = \frac{qV}{\eta_i} \left(\frac{N}{\tau(N)} + v_g g(N)S(N) \right) \quad (2)$$

Equations (1) and (2) are both functions of carrier concentration N . Use N as independent parameter, one can calculate the L-I curve by calculate $S(N)$ and $I(N)$. N ranges from 0 to N_{th} .

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Detailed Analysis: Steady State Solutions (Cont'd)

Below Threshold

$$S(N) \approx 0$$

$$\text{Output power: } P_0 \approx 0$$

$$I(N) = \frac{qV}{\eta_i} \left(\frac{N}{\tau(N)} \right)$$

Above Threshold

$$N \rightarrow N_{th}, \quad g_{th} = g(N_{th}) = \frac{\alpha_i + \alpha_m}{\Gamma}$$

$$\frac{1}{\tau_p} = v_g \cdot (\alpha_i + \alpha_m) = \Gamma v_g g_{th}$$

$$\Rightarrow S(N) = \frac{\beta R_{sp}(N) / v_g}{g_{th} - g(N)}$$

$$I(N) = \frac{qV}{\eta_i} \left(\frac{N_{th}}{\tau(N_{th})} \right) + \frac{qV}{\eta_i} (v_g g(N)S(N))$$

$$= I_{th} + \frac{qV}{\eta_i} (v_g g(N)S(N))$$

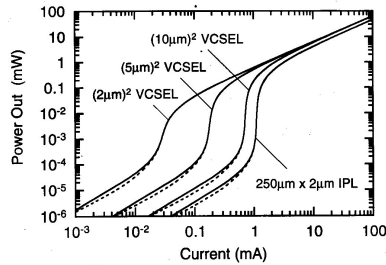
$$P_o = \eta_i \eta_0 \frac{h\omega}{q} (I - I_{th})$$

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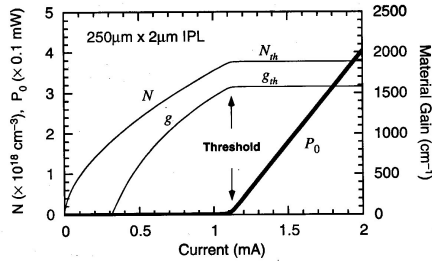
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Steady State (DC) Solutions



- L-I curve in log-log plot
 - Threshold current shows up as “shoulder”
 - Spontaneous emission is clearly seen below threshold



- L-I curve in linear scale
 - Gain is “clamped” at threshold
 - Carrier concentration is also clamped at threshold

FIGURE 5.2 Upper plot: light vs. current in two different lateral-size 3-QW VCSELs and in a single QW in-plane laser (IPL) (all lasers use InGaAs/GaAs 80Å QWs). Lower plot: Light vs. current on a linear scale for the same in-plane laser. Plot also shows carrier density and material gain vs. current.

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Differential Analysis

$$d \left[\frac{dN}{dt} \right] = \frac{\eta_i}{qV} dI - \frac{dN}{\tau_{\Delta N}} - (v_g g dS + v_g S dg)$$

$$d \left[\frac{dS}{dt} \right] = \Gamma (v_g g dS + v_g S dg) - \frac{dS}{\tau_p} + \Gamma \frac{dN}{\tau_{\Delta N}}$$

$$\frac{N}{\tau(N)} = AN + BN^2 + CN^3$$

$$\frac{1}{\tau_{\Delta N}} = A + 2BN + 3CN^2$$

$$\frac{1}{\tau_{\Delta N}} = \frac{d}{dN} (\beta BN^2) = 2\beta BN + \frac{d\beta}{dN} BN^2$$

Gain is generally both a function of N and S :

$$g = g(N, S)$$

$$dg = \frac{\partial g}{\partial N} dN + \frac{\partial g}{\partial S} dS = a \cdot dN - a_p dS$$

Example:

Logarithmic gain model:

$$g(N, S) = \frac{g_0}{1 + \epsilon S} \ln \left(\frac{N}{N_{tr}} \right)$$

$$a = \frac{\partial g}{\partial N} = \frac{g_0}{1 + \epsilon S} \frac{1}{N}$$

$$a_p = -\frac{\partial g}{\partial S} = \frac{\epsilon g}{1 + \epsilon S}$$

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Differential Analysis

The differential rate equations can be expressed as

$$\frac{d}{dt} \begin{bmatrix} dN \\ dS \end{bmatrix} = \begin{bmatrix} -\gamma_{NN} & -\gamma_{NS} \\ \gamma_{SN} & -\gamma_{SS} \end{bmatrix} \begin{bmatrix} dN \\ dS \end{bmatrix} + \frac{\eta_i}{qV} \begin{bmatrix} dI \\ 0 \end{bmatrix}$$

$$\begin{cases} \gamma_{NN} = \frac{1}{\tau_{\Delta N}} + v_g a S \\ \gamma_{NS} = v_g g - a_p v_g S \\ \gamma_{SN} = \Gamma v_g S a \\ \gamma_{SS} = \Gamma v_g S a_p \end{cases}$$

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Small-Signal Analysis

$$dI(t) = I_1 e^{j\omega t}; \quad dN(t) = N_1 e^{j\omega t}; \quad dS(t) = S_1 e^{j\omega t}$$

$$\begin{bmatrix} j\omega + \gamma_{NN} & \gamma_{NS} \\ -\gamma_{SN} & j\omega + \gamma_{SS} \end{bmatrix} \begin{bmatrix} N_1 \\ S_1 \end{bmatrix} = \frac{\eta_i I_1}{qV} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta \equiv \begin{vmatrix} j\omega + \gamma_{NN} & \gamma_{NS} \\ -\gamma_{SN} & j\omega + \gamma_{SS} \end{vmatrix}$$

$$= (\gamma_{NS} \gamma_{SN} + \gamma_{NN} \gamma_{SS}) - \omega^2 + j\omega(\gamma_{NN} + \gamma_{SS})$$

$$= \omega_R^2 - \omega^2 + j\omega\gamma$$

$$N_1 = \frac{\eta_i I_1}{qV} \frac{\gamma_{SS} + j\omega}{\omega_R^2} H(\omega)$$

$$S_1 = \frac{\eta_i I_1}{qV} \frac{\gamma_{SN}}{\omega_R^2} H(\omega)$$

$$H(\omega) = \frac{\omega_R^2}{\Delta} = \frac{\omega_R^2}{\omega_R^2 - \omega^2 + j\omega\gamma}$$

$$\omega_R^2 = \gamma_{NS} \gamma_{SN} + \gamma_{NN} \gamma_{SS}$$

$$\omega_R^2 \approx \frac{v_g a S}{\tau_p} \quad (\text{above threshold})$$

$$\gamma = \gamma_{NN} + \gamma_{SS}$$

$$\gamma = v_g a S \left(1 + \frac{\Gamma a_p}{a} \right) + \frac{1}{\tau_{\Delta N}} + \frac{\Gamma R'_{sp}}{S}$$

$$\gamma = K f_R^2 + \gamma_0$$

$$K = 4\pi^2 \tau_p \left(1 + \frac{\Gamma a_p}{a} \right)$$

$$\gamma_0 \approx \frac{1}{\tau_{\Delta N}}$$

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Frequency Response

Frequency response of the laser:

$$\frac{S_1}{I_1} = \frac{\eta_i \gamma_{SN}}{qV \omega_R^2} H(\omega)$$

$$P_1 = \eta_0 h \omega V v_g g_{th} S_1$$

$$\frac{P_1}{I_1} = \left(\eta_i \eta_0 \frac{h \omega}{q} \right) H(\omega)$$

DC Quantum Efficiency Normalized Frequency Response

$$f_R = \frac{1}{2\pi} \omega_R = \frac{1}{2\pi} \sqrt{\frac{v_g a S}{\tau_p}}$$

f_{3-dB} can be obtained by solving

$$|H(\omega)|^2 = \frac{1}{2}$$

$$f_{3-dB} = f_R \sqrt{1 + \sqrt{2}} \approx 1.55 f_R$$

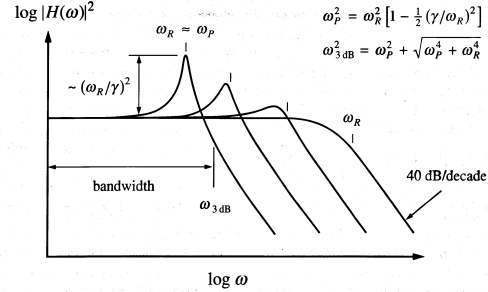


FIGURE 5.4 Sketch of the modulation transfer function for increasing values of relaxation resonance frequency and damping factor, including relationships between the peak frequency, ω_p , the resonance frequency, ω_R , and the 3 dB down cutoff frequency, ω_{3-dB} .

Typical values:

$$v_g \sim 10^{10} \text{ cm/sec}, \quad a \sim 10^{-16} \text{ cm}^2, \quad \tau_p \sim 1 \text{ ps}$$

$$S \sim 10^{15} \text{ 1/cm}^3$$

$$f_R = \frac{1}{2\pi} \omega_R = \frac{1}{2\pi} \sqrt{\frac{v_g a S}{\tau_p}} \sim 5 \text{ GHz}$$



Small-Signal Analysis

3dB frequency:

$$|H(\omega_{3dB})|^2 = \frac{1}{2}$$

$$(\omega_R^2 - \omega_{3dB}^2)^2 + (\omega_{3dB} \gamma)^2 = 2\omega_R^4$$

$$\omega_{3dB}^4 + (\gamma^2 - 2\omega_R^2)\omega_{3dB}^2 - \omega_R^4 = 0$$

$$\omega_{3dB}^2 = \omega_p^2 + \sqrt{\omega_p^4 + \omega_R^4}$$

$$\omega_p^2 = \frac{-(\gamma^2 - 2\omega_R^2)}{2} = \omega_R^2 - \frac{1}{2}\gamma^2$$

Maximum 3dB bandwidth occurs when

$$\frac{d(\omega_{3dB}^2)}{d(\omega_R^2)} = 0$$

$$1 + \frac{1}{2} \frac{2\omega_R^2}{\sqrt{\omega_p^4 + \omega_R^4}} = 0 \Rightarrow \omega_p^4 = 0$$

$$2\omega_{R,max}^2 = \gamma^2 = (Kf_R^2)^2$$

$$f_{3dB,max} = \frac{2\pi\sqrt{2}}{K}$$

$$|H(\omega)|^2 = \frac{\omega_{R,max}^4}{\omega^4 + \omega_{R,max}^4}$$

$$f_{3-dB,max} = f_{R,max} = \frac{2\pi\sqrt{2}}{K}$$